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抛物线形断面渠道收缩水深的解析解

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摘要:通过对抛物线形断面收缩水深的基本方程进行恒等变形,得到的无量纲收缩水深是一个典型的一元四次方程式。根据一元四次方程式的解得到抛物线形断面的收缩水深的解析解表达式,为求解抛物线形断面的收缩水深提供了一种新的解法。该解析公式形式简洁、结果准确、适用范围广,克服了目前查图、查表及试算迭代法的缺点。

关 键 词:抛物线形断面; 收缩水深; 解析解; 渠道
中图分类号:TV131 **文献标识码**:A

1 概 述

抛物线形断面渠道形式由于具有抗冻性能以及水力学条件优越等优点而广泛应用于水利水电、城市给排水以及灌溉排水工程中。目前常用的计算方法就是试算法、图解法,这些方法即费时又费力而且精度不高。张志昌[1]在抛物线形最优断面设计方面进行了较深入研究;卢琴[2]、赵延风[3]等则在计算抛物线形断面渠道收缩水深近似计算方面取得了较好的研究成果。但目前关于抛物线形断面收缩水深的解析解方面尚未见相关的研究报道。为此,本文从抛物线形断面渠道收缩水深的基本方程入手,通过恒等变换,得出的抛物线断面无量纲收缩水深是一个典型的一元四次方程式,通过数学手册即可求解出该方程的解。其表达式非常简洁、结果准确、适用范围广($\lambda \in (0,0.75)$),而且克服了目前查图查表及试算迭代法中计算结果近似和适用范围窄等缺点,极大方便工程设计人员的应用。

2 抛物线形断面收缩水深计算公式

抛物线形渠道过水断面见图 1,设抛物线方程为 $y = Px^2$,P 为形状系数。由抛物线形断面收缩水深的解析公式,收缩水深的基本方程[4]为

$$E_0 = h_c + \frac{Q^2}{2g\phi^2 A_c^2}, \tag{1}$$

式中:E_0 为以收缩断面底部为基准面的泄水建筑物上游总水头(m);h_c 为收缩断面水深(m);Q 为下泄流量(m^3/s);g 为重力加速度(m/s^2);ϕ 为流速系数;A_c 为抛物线形断面面积(m^2)。

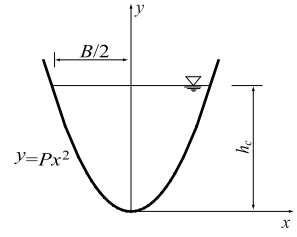


图 1 抛物线形断面图
Fig. 1 Cross-section of parabola-shape

抛物线形断面的过水断面面积为

$$A = \frac{4}{3}P^{-\frac{1}{2}}h^{\frac{3}{2}}。 \tag{2}$$

设无量纲收缩水深

$$\lambda = \frac{h_c}{E_0}, \tag{3}$$

将式(2)、式(3)代入式(1)中,整理则得抛物线形渠道无量纲收缩水深 λ 的计算关系式,

$$\frac{9Q^2P}{32g\phi^2E_0^4} = \lambda^3(1-\lambda)。 \tag{4}$$

为了方便表达,设无量纲参数

$$k = \frac{9Q^2P}{32g\phi^2E_0^4}, \tag{5}$$

则

$$\lambda^4 - \lambda^3 + k = 0。$$

而这个一元四次方程是有解析解的[5],

$$\lambda = -\frac{1}{2}(b + \sqrt{b^2 - 4c})。 \tag{6}$$

式中 a,b,c 为一元四次方程方程式的求解参数,其计算公式如下:

$$a = \sqrt[3]{\frac{k}{16} + \sqrt{\frac{k^2}{256} - \frac{k^3}{27}}} + \sqrt[3]{\frac{k}{16} - \sqrt{\frac{k^2}{256} - \frac{k^3}{27}}}; \tag{7}$$

$$b = -\frac{1}{2}(1 + \sqrt{1 + 8a}); \tag{8}$$

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$$c = a\left(1 + \frac{1}{\sqrt{1+8a}}\right)。\qquad (9)$$

3　公式适用范围及精度

抛物线断面无量纲收缩水深 λ 的取值范围：如果仅从数学分析上看来，它的定义域应为 $\lambda \in (0,1)$；结合水力学分析，由于断面收缩水深总是小于断面临界水深，而抛物线形断面的临界水深

$$h_k = \left(\frac{27P\varphi Q^2}{32g}\right)^{\frac{1}{4}},$$

代入基本方程得到临界流时的无量纲水深

$$\lambda_k = \frac{1}{1+\frac{1}{3\phi^3}},$$

因流速系数 $\phi < 1$ 得临界流时 $\lambda_k < 0.75$，因此得出 $0 < \lambda < \lambda_k < 0.75$；而在实际工程中无量纲收缩水深常用范围为 $\lambda \in (0,0.5)$。

由于公式(6)是解析解，所以其计算结果是没有任何误差的。

4　计算步骤及应用举例

4.1　计算收缩水深步骤

(1) 根据断面前总水头 E_0、通过流量 Q、流速系数 ϕ 和抛物线形断面形状系数 P，由式(5)求参数 k；

(2) 由式(7)、(8)、(9)分别求一元四次方程式的求解参数 a,b,c；

(3) 由式(6)求抛物线断面的无量纲收缩水深 λ；

(4) 由式(3)求得抛物线断面的收缩水深 h_c。

4.2　计算举例

为了便于比较，我们仍采用文献[3] 中的算例。已知某坝前断面总水头 $E_0 = 10.31$ m，通过流量 $Q = 140\,\mathrm{m^3/s}$，流速系数 $\phi = 0.95$，若采用方程为 $y = 0.25x^2$ 的抛物线形断面(见图2)，求坝下断面收缩水深 h_c。

解：①根据已知条件由式(5)得参数 $k = 0.013\,78$；② 由式(7)、(8)、(9)分别求一元四次方程式的求解参数 $a = 0.157\,25$，$b = -1.251\,33$，$c = 0.261\,90$；③由式(6)求无量纲收缩水深；④由式(3)求得收缩水深 $h_c = 2.739\,58$。

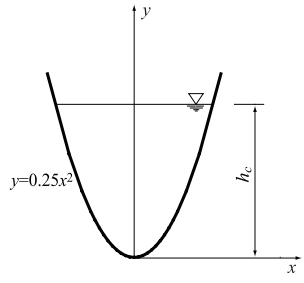


图 2　$y = 0.25x^2$ 的抛物线形断面图

Fig. 2　Cross-section of parabola $y = 0.25x^2$

5　结　语

本文的解是从收缩水深方程直接入手，推导过程中不含任何近似假设，因此解答是精确的，且适用范围较广，即无量纲收缩水深在 $\lambda \in (0,0.75]$ 范围内均可适用，而工程常用范围为 $\lambda \in (0,0.5]$。为抛物线形断面收缩水深的计算提供了一种新的方法。

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Analytical Solution of Water Depth in Parabolic-Shaped Channel with Contracted Section

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Abstract: Through the identical deformation of the basic equation about the contracted water depth of parabolic-shaped channel, we can obtain a non-dimensional formula, which is a typical quartic equation with one unknown . According to this equation's solution, we can also obtain an analytical expression for calculating water depth about

the contracted water depth of parabolic-shaped channel, it provides one kind of new solution for calculating the water depth about the contracted section of parabolic-shaped channel. The analytical formula is short-cut and its result is correct, and can be widely used. It also overcomes the shortage of consulting graph-list and trial computation methods widely used in the past-time.

Key words: parabolic-shaped section; contracted water depth; analytical solution; channel

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Application of Pattern Search Optimization in Calculation of Normal Depth in Trapezoidal Canal

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Abstract: Normal depth is an important parameter occurring in the design of irrigation canals. Direct analytic solution of normal-depth problems is not possible, as the governing equations for the practical canal sections are implicit. The solution requires tedious methods of trial and error. Tabular and graphical methods also available for solution are subject to errors of double interpolation and errors of judgment in reading the graphs. In order to improve this situation, a new formula is derived from transforming identically the uniform flow equation in trapezoidal canal through introducing a non-dimensional parameter-unit surface width. Also, it is proved to be monotonic function. So, the calculation of normal depth problem was converted into a nonlinear constrained optimization problem, which can be solved by the pattern search optimization method. Finally, Error analysis and an illustration computed using this method show that it is much more applicable, precise and simple than tradition methods. So it provided a new tool for obtaining normal depth of open channel in trapezoid section problem.

Key words: normal depth; pattern search algorithm; trapezoidal canal; unit surface width

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Experimental Analysis and Numerical Simulation of U-shape Bend Flow

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Abstract: The structure of an U-shape bend flow including lateral slope of water surface, streamwise velocity, circulation flow and typical *u-v* vectors along water depth directions were analyzed. Velocities were measured using up-looking and down-looking Sontec-ADV (acoustic doppler velocimenters). Furthermore, the numerical modeling of bend flow has also been conducted in the experimental flume under various flow conditions. These simulated results agree well with the experimental data.

Key words: bend flow; experimental analysis; numerical simulation; streamwise velocity; circulation flow